

Abstract Algebra I

Problem Set 2: Symmetric Groups

Exam style questions are marked with (*).

Groups

Question 1. Verify that the set of permutations of n elements $\{1, \dots, n\}$ is a group with permutation composition. (Verify the group axioms.)

Question 2. Verify that the set of symmetric transformations of a regular n -simplex is a group with composition of transformations. (Verify the group axioms.)

Question 3. Find the number of elements of the groups mentioned in Questions (1) and (2).

Question 4. (*) Show that $|S_n| = 2|A_n|$. Show that $A_n < S_n$. Conclude that $A_n \trianglelefteq S_n$.

Question 5. Relate your intuitive understanding of orientation to the definition from class, i.e. that the orientation of a labelling of vectors $v_1, \dots, v_n$ in $\mathbb{R}^n$ is the sign of the determinant of the matrix whose columns are the vectors v_i :

$$\operatorname{sgn} \left(\det \begin{pmatrix} v_1 & v_2 & \dots & v_n \end{pmatrix} \right).$$

Challenging

Question 6. Show that the group of permutations of n elements is the group of symmetric transformations of the regular $(n+1)$ -simplex.